Anomalous Crossing Frequency in Odd Proton Nuclei

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Abstract

A generic explanation for the recently observed anomalous crossing frequencies in odd proton rare earth nuclei is given. As an example, the proton $\frac{1}{2}[541]$ band in ^{175}Ta is discussed in detail by using the angular momentum projection theory. It is shown that the quadrupole pairing interaction is decisive in delaying the crossing point and the changes in crossing frequency along the isotope chain are due to the different neutron shell fillings.

Over twenty years ago, Bes and Broglia introduced the quadrupole pairing force in their particle-vibration-coupling model [1]. It was shown later that such a force will tend to attenuate the Coriolis interaction in an odd mass system [2,3] and shift the band crossing point in an even-even system [4,5]. Curiously the effect of this force on the odd mass band crossing did not receive much attention. The lack of which, even to this date, may be due partly to the fact that there is no relevant data appropriate for this investigation. In the past few years, a large amount of high spin data were obtained for odd proton isotopes (Lu, Ta, Re and Ir) and they systematically showed an anomalous shift in the crossing frequency (see [6,7] and references therein). Quite recently, an especially large crossing frequency for the proton band labeled by the Nilsson quantum numbers $\frac{1}{2}[541]$ in ^{175}Ta was observed at Beijing's Institute of Atomic Energy Tandem Laboratory [8]. In addition, it was also observed that going from light to heavy Ta isotopes, there is a drastic change in the crossing frequency. With this set of new data, the situation is riped to carry out a study of the above mentioned problem. In this letter we shall discuss the physics mainly by using ^{175}Ta as an example. However, it is worth emphasizing that the discussions could be equally applied to other odd proton nuclei as well.

It is well known that in rare earth nuclei, the crossing of bands (i.e. two bands with rather different rotational frequencies cross at a certain angular momentum) can be interpreted as the alignment of a pair of $i_{\frac{13}{2}}$ quasineutrons along the rotational axis. For ^{175}Ta the crossing frequency was found to be 0.375 MeV/$\hbar$. This significant increase in value from its even-even neighbors (0.292 MeV/$\hbar$ for ^{174}Hf and 0.291 MeV/$\hbar$ for ^{176}W) indicates that there must be a physical mechanism to delay the crossing of the bands. Indeed the problem is made more acute when Wen et al. [8] found that within the framework of the semi-classical cranking shell model, one needs to use an unreasonably large quadrupole deformation ϵ_2 of 0.3 to account for this observation. However, there is a lingering question as to whether the cranking theory is useful to discuss the phenomena of band crossing since in the cranking approximation it admixes bands at a given rotational frequency (not at a given angular momentum), thus giving rise to large uncertainty in the band crossing region [9].

Clearly, the study of the problem requires a model which can treat the band crossings quantum mechanically. Shell model configuration mixing calculations could in principle solve this problem, but in practice is unfeasible for heavy systems. The angular momentum projection theory established in the late seventies [10,5] has been demonstrated to be a powerful model to quantitatively account for many high spin phenomena [11–13]. The development of an efficient algorithm [10] renders it possible, in a unified manner, to systematically investigate the even-even, odd-odd and odd mass heavy systems. Hence the model is particularly suitable for the present study. Since the model has already been extensively discussed [5,11], only the salient features will be given below. We should add that a similar approach to the projection technique was also developed by the Tübingen group [14].

The ansatz for the angular momentum projected wave function is given by

$$|IM> \ = \ \sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa} |\varphi_\kappa>, \tag{1}$$

where κ labels the basis states. Acting on an intrinsic state $|\varphi_\kappa>$, the projection operator $\hat{P}^I_{MK}$ [15] generates states of good angular momentum, thus restoring the necessary rotational symmetry which was violated in the deformed mean field. The advantage of the present approach is that the crossing and mixing of bands at a given angular momentum are treated fully quantum mechanically. This turns out to be crucial to treat the present problem.

In the present work, we assume that the intrinsic states have axial symmetry. Thus, the basis states $|\varphi_\kappa>$ must have K as a good quantum number. Since the nuclei in question have only very weak γ deformation, this restriction shall not prevent us to investigate the physics at hand. The basis states $|\varphi_\kappa>$ are spanned by the set

$$\left\{ \ \alpha^\dagger_{p_l}|\phi>, \quad \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}\alpha^\dagger_{p_l}|\phi> \ \right\}$$
$$\left\{ \ |\phi>, \quad \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}|\phi>, \quad \alpha^\dagger_{p_k}\alpha^\dagger_{p_l}|\phi>, \quad \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}\alpha^\dagger_{p_k}\alpha^\dagger_{p_l}|\phi> \ \right\} \tag{2}$$

for odd proton and even-even nuclei, respectively. The quasiparticle vacuum is $|\phi>$ and $\{\alpha_m, \alpha^\dagger_m\}$ are the quasiparticle annihilation and creation operators for this vacuum; the index n_i (p_i) runs over selected neutron (proton) quasiparticle states and κ in eq. (1)

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runs over the configurations of eq. (2). The vacuum $|\phi>$ is obtained by diagonalizing a deformed Nilsson hamiltonian [16] followed by a BCS calculation. In the calculation, we have used three major shells: i.e. N = 4, 5 and 6 (N = 3, 4 and 5) for neutrons (protons) as the configuration space. For the odd system, the BCS blocking effect associated with the last unpaired proton is approximately taken into account by allowing all the odd number of protons to participate without blocking any individual level. Thus the vacuum in this case is an average over the two neghboring even-even nuclei. The size of basis states, which includes the most important configurations, is determined by using energy windows of 1.5 MeV, 2.5 MeV, 4 MeV and 5 MeV for the 1-, 2-, 3- and 4-qp states, respectively.

In this work, we have used the following hamiltonian [5]

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_\mu \hat{Q}_\mu^\dagger \hat{Q}_\mu - G_M \hat{P}^\dagger \hat{P} - G_Q \sum_\mu \hat{P}_\mu^\dagger \hat{P}_\mu, \tag{3}$$

where $\hat{H}_0$ is the spherical single-particle shell model hamiltonian. The second term is the quadrupole-quadrupole interaction and the last two terms are the monopole and quadrupole pairing interactions respectively. The interaction strengths are determined as follows: the quadrupole interaction strength χ is adjusted so that the known quadrupole deformation ϵ_2 from the Hartree-Fock-Bogoliubov self-consistent procedure [17] is obtained. For example, for ^{175}Ta it is 0.26; the monopole pairing strength G_M is adjusted to the known energy gap

$$G_M = \left[20.12 \mp 13.13\frac{N-Z}{A}\right] \cdot A^{-1}, \tag{4}$$

where the minus (plus) sign is for neutrons (protons). The quadrupole pairing strength G_Q is assumed to be proportional to G_M and the proportional constant is typically $C \approx 0.20$. This is the only adjustable parameter in the present model.

The weights f_κ in eq. (1) are determined by diagonalizaing the hamiltonian $\hat{H}$ in the basis given by of eq. (2). This will lead to the eigenvalue equation (for a given spin I)

$$\sum_{\kappa'}(H_{\kappa\kappa'} - EN_{\kappa\kappa'})f_{\kappa'} = 0, \tag{5}$$

with the hamiltonian and norm overlaps given by

$$H_{\kappa\kappa'} = <\varphi_\kappa|\hat{H}\hat{P}^I_{K_\kappa K'_{\kappa'}}|\varphi_{\kappa'}>,$$

$$N_{\kappa\kappa'} = <\varphi_\kappa|\hat{P}^I_{K_\kappa K'_{\kappa'}}|\varphi_{\kappa'}> . \tag{6}$$

Projection of good angular momentum onto each intrinsic state generates the rotational band associated with this intrinsic configuration $|\varphi_\kappa>$. For example, $\hat{P}^I_{MK}\alpha^\dagger_{p_l}|\phi>$ will produce a one-quasiproton band. The energies of each band are given by the diagonal elements of eq.(6)

$$E_\kappa(I) = \frac{<\varphi_\kappa|\hat{H}\hat{P}^I_{KK}|\varphi_\kappa>}{<\varphi_\kappa|\hat{P}^I_{KK}|\varphi_\kappa>} = \frac{H_{\kappa\kappa}}{N_{\kappa\kappa}}. \tag{7}$$

A diagram in which $E_\kappa(I)$ of various bands are plotted against the spin I will be referred to [11] as a band diagram. It will reveal information for understanding the character of the observed band crossings. The results obtained from diagonalizing the hamiltonian of eq.(3) can be compared with the experiments.

In fig.1, the band diagram for negative parity bands of ^{175}Ta is presented. Although there are many bands in the calculation, only four most interesting bands will be plotted in order to illustrate the main features. The rotational frequency of each band, $\omega(I) = \frac{dE(I)}{dI}$, is naturally described by the slope of the curve. Its inverted value gives the moment of inertia. In fig.1, one can see that at a certain angular momentum, different configurations have different slopes. The band which is labeled by the Nilsson quantum numbers $\frac{9}{2}[514]$ shows the usual smooth behavior as a function of increasing angular momentum. From the figure, we see that it roughly crosses with the $\frac{1}{2}[541]$ band at spin $\frac{21}{2}\hbar$ and continues upward monotonously. At about spin $\frac{31}{2}\hbar$, it enters into a region where several bands converge and interact with each other. At this point, the experimental assignments [8] of the levels can no longer be made in a clear cut manner. We predict that the $\frac{9}{2}[514]$ band will cross the 3-qp band at spin $\frac{37}{2}\hbar$. We anticipate that one should be able to observe this band crossing if the present data [8] for the $\frac{9}{2}[514]$ band is extended to higher spins.

For the $\frac{1}{2}[541]$ band, the zig-zag behavior indicates a strong signature splitting in energy. In fact, only the favored branch with signature $\frac{1}{2}$ has been experimentally observed. This

one-quasiproton band clearly crosses the 3-qp band at spin $\frac{45}{2}\hbar$, thus producing the observed anomaly in the spectra. After this crossing, the structure of the Yrast band should mainly be 3-qp in nature. It should be pointed out that without any additional assumption, our calculation clearly indicates that the $\frac{1}{2}[541]$ band crosses the 3-qp band at a much later stage (spin $\frac{45}{2}\hbar$) than the $\frac{9}{2}[514]$ band (spin $\frac{37}{2}\hbar$).

In fig.2a we compare the results after diagonalization with the data [8] for the proton $\frac{1}{2}[541]$ band. This can be succinctly presented by plotting the rotational frequency as a function of the angular momentum. As can be seen from this figure, the theory agrees well with the data. In particular, the rotational alignment at spin $\frac{45}{2}\hbar$ is reproduced. It is important to notice that the quadrupole pairing force in the hamiltonian of eq. (3) is crucial here. Its influence on the results is demonstrated in fig.2b. Indeed, by increasing the C from 0.16 to 0.24, a clear delay of the alignment process is obtained. If this force is not included in the hamiltonian (i.e. $C = 0$), then the alignment could occur as early as spin $\frac{33}{2}\hbar$. The physical reason behind the delay is as follows: If a zero angular momentum pair is broken in the absence of quadrupole pairing, then there exists no additional force to resist the alignment process beyond that point [4]. In other words, the quadrupole pairing interaction prevents the alignment from occuring too soon. Suffice to mention here that the positive parity bands for ^{175}Ta have also been computed and they too agree well with the data for all the known bands [8]. Detail discussions will be published elsewhere.

It is natural to inquire whether the drastic changes in the crossing frequency along the Ta isotope chain requier the readjustment of the quadrupole pairing force along the chain since this could be a judgement of the present model. It is therefore gratifying that without changing any of the parameters (in particular, keeping $C = 0.24$), we can reproduce the observed crossing frequencies of the isotopes ^{167,169,171,173}Ta, as shown in fig.3. We should mention that for the two lighter isotopes ^{167}Ta and ^{169}Ta, although the crossing positions are correctly predicted, there is too much alignment. In addition, for ^{169}Ta and ^{171}Ta, the deviation between theory and data at high spins from $\frac{57}{2}\hbar$ onwards can be attributed to the missing configurations of the 5-qp states in the computer code. One believes that crossings

between 3-qp and 5-qp bands can occur here [6].

The different crossing frequencies for various isotopes are simply due to the neutron shell fillings. We know that the proton Fermi level remains nearly identical for all the isotopes. Therefore, the energy and the character of the projected one-quasiproton state should remain unchanged. However, because of the differences of the neutron Fermi levels, the energies and the configurations of the additional neutron pair in the projected 3-qp states can and will change with neutron number. This can alter the behavior along the isotopic chain of the 3-qp bands in the energy versus angular momentum band diagram. In fact, they can differ in energies (the bands lie higher or lower) and/or rotational frequencies (the slope of bands are steeper or flatter). Consequently the crossing positions between the proton $\frac{1}{2}[541]$ and the 3-qp bands and their interactions are quite different. Thus in hindsight it is not surprising that there are drastic changes in the crossing frequencies along the chain.

As we have mentioned at the beginning, the delay of the crossing frequency is measured by comparing an odd mass nucleus with its even-even neighbors. Hence a unified treatment will demand us to examine the even-even neighboring nuclei with the same theory as well. In fig.4, we present our results for ^{174}Hf and ^{176}W. By varying the quadrupole pairing strength, one observes the effect of shifting the crossing points. However, the effect is clearly not as significant as in the odd mass Ta case. It seems that the quadrupole pairing force is much less sensitive to the even system. Hence the theory without any re-adjustment in the interacting strength, can consistently describe band crossings for various systems. Furthermore, we notice that the effect is less pronounced at very low spins than it at higher spins, as one can see from fig. 2b. Further investigations of this point is clearly necessary.

In conclusion, we have studied the anomalous crossing frequency observed in $\frac{1}{2}[541]$ band in the Ta isotopes. It has been shown that by using the projection theory and including the quadrupole pairing interaction in the hamiltonian, one can reproduce the essential physics here. In particular the crossing frequencies of all five Ta isotopes, especially the delay in alignment in ^{175}Ta can be described in a unified manner and is a natural consequence of certain neutron shell filling. The influence of the quadrupole pairing force on the isotons

^{174}Hf and ^{176}W has also been discussed. This is an example of how one could amalgamate in a unified manner in one system the two seemingly unrelatated effects of the quadrupole pairing force, namely attenuating the Coriolis force and shifting the band crossings found in early studies. Furthermore, from this study it suggests that the high spin region may just be the sensitive window to determine accurately one of the effective interactions, i.e. the quadrupole pairing interaction.

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FIGURES

FIG. 1. Band diagram of ^{175}Ta. Two proton 1-qp bands $\frac{9}{2}[514]$ and $\frac{1}{2}[541]$ and the corresponding 3-qp bands are plotted. In the calculation, the quadrupole pairing strength $C = 0.24$ is used.

FIG. 2. Rotational frequency versus angular momentum plot for the proton $\frac{1}{2}[541]$ band in ^{175}Ta. a) Top: Comparison of the calculation with data [8]. b) Bottom: Influence of the quadrupole pairing force on the crossing frequency.

FIG. 3. Rotational frequency versus angular momentum plot for the proton $\frac{1}{2}[541]$ band for four Ta isotopes. Data are taken from: ^{167}Ta [18], ^{169}Ta [6], ^{171}Ta [19] and ^{173}Ta [19]. In the calculation, the quadrupole pairing strength $C = 0.24$ is used.

FIG. 4. Rotational frequency versus angular momentum plot for the yrast band of the two isotons ^{174}Hf and ^{176}W. Data are taken from: ^{174}Hf [20] and ^{176}W [21]. The influence of the quadrupole pairing force on the crossing frequency are shown.

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